

In the *Nouveau Bulletin des Sciences, par la Société Philomathique de Paris*, tome ii., Paris, 1810, occurs this passage:—

“*Mathématiques.*—Sur les Équations différentielles des Courbes du Second Degré, par M. Monge. L'équation générale des courbes du second degré étant

$$Ay^2 + 2Bxy + Cx^2 + 2Dy + Ex + 1 = 0,$$

dans laquelle  $A, B, C, D, E$  sont des constantes, M. Monge donne l'équation différentielle débarrassée de toutes ces constantes, et il parvient à l'équation suivante, du cinquième ordre,

$$(A) \quad 9g^2t - 45qrs + 40r^3 = 0,$$

les quantités  $r, s, t$ , étant définies par les équations suivantes :

$$\frac{dy}{dx} = p, \quad \frac{dp}{dx} = q, \quad \frac{dq}{dx} = r, \quad \frac{dr}{dx} = s, \quad \frac{ds}{dx} = t.$$

“Il faut ensuite voir l'usage de l'équation (A), pour trouver l'intégrale d'une équation d'un ordre inférieur qui satisfait à cette équation (A); ainsi étant donnée l'équation différentielle  $(1 + p^2)r = 3pq^2$ , il parvient à l'intégrale  $(x - a)^2 + (y - b)^2 = c^2$ , qui est l'équation d'un cercle.

“La même méthode pourroit s'appliquer aux équations des courbes d'un degré supérieur au second.”

A note is added to the effect that “Cet article est extrait de la Correspondance de l'École impériale Polytechnique, rédigée par M. Hachette: 1<sup>er</sup> cahier du 2<sup>e</sup> volume, 1810.” The press mark of this work at the British Museum is PP. 1543.

Trusting that this is the reference you are in search of, and that the long delay in the discovery of it may be excused when the difficulty of identifying a particular passage (known perhaps only in its full extent to those whose chief work is concerned with such matters) is considered.

I remain, Sir, faithfully yours,

H. FISHER

PROF. J. J. SYLVESTER, &c., &c.

New College, Oxford, April 19

J. J. SYLVESTER

### On the Velocity of Light as Determined by Foucault's Revolving Mirror

It has been shown by Lord Rayleigh and others that the velocity ( $U$ ) with which a group of waves is propagated in any medium may be calculated by the formula—

$$U = V \left( 1 - \frac{d \log V}{d \log \lambda} \right),$$

where  $V$  is the wave-velocity, and  $\lambda$  the wave-length. It has also been observed by Lord Rayleigh that the fronts of the waves reflected by the revolving mirror in Foucault's experiment are inclined one to another, and in consequence must rotate with an angular velocity—

$$\frac{dV}{d\lambda} \alpha,$$

where  $\alpha$  is the angle between two successive wave-planes of similar phase. When  $dV/d\lambda$  is positive (the usual case), the direction of rotation is such that the following wave-plane rotates towards the position of the preceding (see *NATURE*, vol. xxv. p. 52).

But I am not aware that attention has been called to the important fact, that while the individual wave rotates the wave-normal of the group remains unchanged, or, in other words, that if we fix our attention on a point moving with the group, therefore with the velocity  $U$ , the successive wave-planes, as they pass through that point, have all the same orientation. This follows immediately from the two formulæ quoted above. For the interval of time between the arrival of two successive wave-planes of similar phase at the moving point is evidently  $\lambda/(V - U)$ , which reduces by the first formula to  $d\lambda/dV$ . In this time the second of the wave-planes, having the angular velocity  $dV/d\lambda$ , will rotate through an angle  $\alpha$  towards the position of the first wave-plane. But  $\alpha$  is the angle between the two planes. The second plane, therefore, in passing the moving point, will have exactly the same orientation which the first had. To get a picture of the phenomenon, we may imagine that we are able to see a few inches of the top of a moving carriage-wheel. The individual spokes rotate, while the group maintains a vertical direction.

This consideration greatly simplifies the theory of Foucault's experiment, and makes it evident, I think, that the results of all

such experiments depend upon the value of  $U$ , and not upon that of  $V$ .

The discussion of the experiment by following a single wave, and taking account of its rotation, is a complicated process, and one in which it is very easy to leave out of account some of the elements of the problem. The principal objection to it, however, is its unreality. If the dispersion is considerable, no wave which leaves the revolving mirror will return to it. The individual disappears, only the group has permanence. Prof. Schuster, in his communication of March 11 (p. 439), has nevertheless obtained by this method, as the quantity determined by “the experiments hitherto performed,”  $V^2/(2V - U)$ , which, as he observes, is nearly equal to  $U$ . He would, I think, have obtained  $U$  precisely, if for the angle between two successive wave-planes of similar phase, instead of  $2\pi\lambda/V$ , he had used the more exact value  $2\pi\lambda/U$ .

By the kindness of Prof. Michelson, I am informed with respect to his recent experiments on the velocity of light in bisulphide of carbon that he would be inclined to place the maximum brilliancy of the light between the spectral lines D and E, but nearer to D. If we take the mean between D and E, we have—

$$\frac{K}{U} = 1.745, \quad \frac{K(2V - U)}{V^2} = 1.737,$$

$K$  denoting the velocity in *vacuo* (see *Amer. Jour. Sci.*, vol. xxxi. p. 64). The number observed was 1.76, “with an uncertainty of two units in the second place of decimals.” This agrees best with the first formula. The same would be true if we used values nearer to the line D.

J. WILLARD GIBBS

New Haven, Connecticut, April 1

### The Effect of Change of Temperature on the Velocity of Sound in Iron

I VENTURE to draw attention to an error relating to the above subject, which, originating with Wertheim, still holds a place in some of our modern books on science. According to Wertheim, the velocity of sound in iron and steel is *increased* by rise of temperature not extending beyond 100° C. Now in no sense whatever is this statement correct. It is true that the longitudinal elasticity of iron, as determined by the static method, will be found greater at 100° C. than at 0° C., provided we begin with the lower temperature first and the wire has not, after the original annealing, been previously raised to 100° C.; but the apparent temporary increase of elasticity is really a permanent one (*Phil. Trans.*, part i., 1883, pp. 129–131), and if the wire be repeatedly heated to 100° C. and afterwards cooled, subsequent tests will always show a *less* elasticity at the higher temperature than at the lower, if sufficient rest after cooling be allowed. When, however, we come to such small molecular displacements as are involved in the passage of sound through a wire, even the apparent increase of elasticity mentioned above vanishes. I have been able to prove that, when an iron or steel wire is thrown into longitudinal vibrations, so as to produce a musical note, the pitch of this note becomes lower as we raise the temperature, even when the wire is heated for the first time after it has left the maker's hands.

It seems rather strange that this error should have so long been allowed to remain uncorrected, for it has been known for many years that the pitch of a tuning-fork made of steel is lowered by small rises of temperature, and the main part of this lowering must be due to the decrease of elasticity of the steel.

HERBERT TOMLINSON

King's College, Strand, April 10

### Sound-producing Apparatus of the Cicadas

WITH regard to the above subject, treated of in an article by Mr. Lloyd Morgan in February last (*NATURE*, February 18, p. 368), I may mention that some time ago I examined the drum of the common cicadas found plentifully in the Himalaya near Simla, and which in the evenings send forth a deafening roar from the rhododendron-trees like the whirr of large machinery. Generally the arrangement of the drum and the powerful muscles was as figured by Mr. Morgan, but I also noticed the following particulars not mentioned by him.

The chitinous rods in the membrane of the drum were not parallel, but converged slightly towards one point of the mem-

brane. The effect of this when the sound-producing motion set in was to cause the membrane to wrinkle sharply towards the point of convergence; and, by experiment on the dead insect with the point of a pencil, it was easy to see that the sound was simply produced by this sharp wrinkling of the membrane. If a piece of stiff paper or parchment be held in the fingers, and the thumb be made to play sharply and rapidly upon it in succession, so as to produce a "kink" or wrinkle each time, a very fair representation of the sound of the insect will be produced. A captive insect, when the motion is slowing down, can be advantageously watched; it will then be seen that, as the sound divides up into separate clicks, the membrane becomes alternately wrinkled and flat. Beyond doubt the sound is no humming.

C. S. MIDDLEMISS

North-West Himalaya, March 14

### Ferocity of Animals

I HAVE read with interest the article by Prof. Lloyd Morgan "On the Study of Animal Intelligence" in the present number of *Mind*, in which he touches upon the subject of entangling fact and inference which attracted my attention when reading "Mental Evolution in Animals" some time since.

I write to call Prof. Morgan's attention to the excellent example of "ejective inference" given by Dr. Romanes in his letter in *NATURE* for April 1 (p. 513), where he says of a rat that he "perfectly understood my object." Would it be troubling Dr. Romanes too much to ask him to explain the appearance a *wild* rat presents on "perfectly well understanding" the object of a human being's actions?

Churchfield, Edgbaston, April 5

F. H. COLLINS

### Tropical Dew

HAVING had occasion to lay out a large quantity of iron hoes and picks, without handles, on the hard ground of an open inclosure in one of the driest districts in India (Bellary), where, in fact, these implements had been collected in the face of a scarcity, it was found, after they had lain a couple of months, that a thick, weedy, but luxuriant vegetation had sprung up, enough, though there had been no rain, to almost hide the tools.

The effect depositing tools on grass has had in stimulating its growing the writer has observed in the tropics before, but was at a loss to account for it, except upon some irresolvable theory of radiation or magnetism.

The whole phenomenon is cleared up by Mr. Aitken's paper on "Dew" in *NATURE* of January 14 (p. 256), dew being proved deposited, not, as generally thought, from the air above, but rising and condensing from the soil below; and the ground in India is always hygroscopic. The outer surfaces of the iron tools radiate of course quickly at night, and the stratum of air inclosed between the metal under surfaces and the earth is therefore saturated with condensing moisture.

That iron gratings laid on bare ground will raise a rank vegetation in places with only 10 or 15 inches of annual rainfall, and exposed to tropical heat, is a not unimportant fact, as being a readily available substitute for irrigation water, worth further investigation.

A. T. FRASER

India, March 26

### The Climbing Powers of the Hedgehog

I AM advised by some of my friends to send you a notice of the mode in which hedgehogs may frequently escape from confinement, and of their habits.

I obtained a hedgehog last week, and put it in my kitchen. Every day it is placed in a small back area, about 12 feet square, during the day-time. The waste-pipes from the cisterns discharge into this area, and the animal frequently lies under these, and, as my servant says, "wallows in the trough like a pig." If he hears any noise he at once runs to a corner and rolls himself up.

On Wednesday the servant found him on the top of the partition wall between my area and the next. This wall is vertical, height 9 feet 6 inches. The top course but one projects 1 inch, so he must have climbed over this.

He has been watched in the operation. He climbs by the projecting mortar beds, which are rather rough, looking about him frequently to see if he is watched. He climbs up the house wall beside the pipe in the corner—an ordinary iron rain-pipe; but from

the shoulder of the pipe, where it passes through the wall, to the top of the partition wall, there is a distance of 9 inches without any pipe, so up this portion and over the projecting brick course he must have climbed by clinging to the wall of the house or the partition wall.

Yesterday (Thursday) he repeated the ascent, and descended into the next area, where he was found this morning.

ROBERT H. SCOTT

6, Elm Park Gardens, April 16

### STARS WITH BANDED SPECTRA<sup>1</sup>

THE spectroscopic survey of the northern heavens, undertaken conjointly by MM. Vogel and Dunér in 1879, has already progressed so far that its general results can be fairly anticipated—its immediate results, that is to say; for it is ultimately designed, not so much for a collection of statistics, however valuable and interesting, as for a criterion of change. This effect, however, must wait for the future—perhaps a remote future—to develop; we can in the meantime gather much present knowledge through labours inspired by still unfolded possibilities.

The first instalment of the first spectroscopic star-catalogue systematically executed, was published by Vogel in 1883 (*Publicationen des astrophysikalischen Observatoriums zu Potsdam*, No. 11). It covers a zone of the heavens extending from  $-2^{\circ}$  to  $+20^{\circ}$  of declination, and includes 4051 stars down to 7.5 magnitude. M. Dunér now sends us from Lund, in a catalogue of 352 stars fully ascertained to possess spectra of the fluted and zoned types, a work of special and extreme importance.

Stars with banded spectra fall into two perfectly distinct classes, of which the first is well exemplified in  $\alpha$  Orionis (Betelgeux), the second in a 5.5 magnitude star close behind the Great Bear, numbered 152 in Schjellerup's Catalogue of Red Stars (*Astr. Nach.*, No. 1591), and called by Father Secchi "La Superba," from the extraordinary vivacity of its prismatic rays. The spectrum of Betelgeux (Fig. 1) shows a series of seven or eight well-marked dark bands (besides minor shadings) all abruptly terminated towards the violet, and dying out by insensible gradations towards the red. The impression upon the eye resembles that of a colonnade thrown into strong relief by a vivid side-illumination. Only three conspicuous dark spaces, on the other hand, interrupt the beams of 152 Schjellerup (Fig. 3); but their breadth is fully twice that of the flutings in the spectrum of  $\alpha$  Orionis; and, still more remarkable, they *face in the opposite direction*. Their obscurity deepens slowly downwards towards their less refrangible sides, then suddenly, by a sharp transition, and with a singular and splendid effect of contrast, gives place to unclouded light.

The stars characterised by these two different qualities of absorption, respectively constituted Father Secchi's third and fourth spectral orders. M. Vogel, however, saw fit in 1874 (*Astr. Nach.*, No. 2000) to modify the arrangement by grouping the two varieties together as subdivisions of a single class. Nor was this a mere arbitrary change. It was the outcome of a far-reaching speculation regarding the course of development taken by the great army of suns marshalled in the profundities of space.

Secchi's classification involved no hypothesis of any kind; it was founded simply on appearances. But the idea that the colours, consequently the spectra of stars, may guide us to a knowledge of their comparative "ages," thrown out in a crude shape by Zöllner in 1865, had, meantime, made its way. Vogel's adoption of it as a means of rationalising observed particulars, gave it (perhaps prematurely) a recognised scientific status.

According to this view, the white stars forming Secchi's first order (of which Sirius and Vega may be taken as

<sup>1</sup> "Sur les Étoiles à Spectres de la Troisième Classe." Par N. C. Dunér. Mémoire présenté à l'Académie Royale des Sciences de Suède, le 11 Juin, 1884. (Stockholm, 1884.)